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ANALYTICAL SOLUTION OF D DIMENSIONAL DIRAC EQUATION WITH Q-DEFORMED TRIGONOMETRIC SCARF POTENTIAL FOR EXACT SPIN SYMMETRY USING ROMANOVSKI POLYNOMIAL

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ABSTRACT

The bound state solutions of D dimensional Dirac equation for q-deformed trigonometric Scarf potential under excat spin symmetric limit is investigated using Romanovski polynomial method. The Dirac equations in D dimension for q-deformed Scarf potential in the exact spin symmetric case reduces to Schrodinger type equation for shape invariant potential. In the approximation scheme of centrifugal term, the D dimensional relativistic wave functions are expressed polynomial form and the D dimensional relativistic energy spectra are numerically calculated using Mat Lab from relativistic energy equation.

Key Word: D dimensional Dirac equation, q-deformed Scarf potential, excact spin symmetry, Romanovski polynomial

INTRODUCTION

Relativistic Quantum Mechanic plays important roles in some area of physics. Finding an accurate exact solution of Dirac equation for a certain potential is one of its important roles. The bound state solutions of Dirac equation for some potentials, in the case of spin and pseudo spin symmetries, have been investigated using NU method [1-6], SUSY QM method [7-11], and Romanovski polynomial method [12-16] by some authors. The spin symmetry occurs when the different between repulsive vector potential with the attractive scalar potential is equal to constant, while the pseudospin symmetry arises when the sum of the scalar potential with vector potential is equal to constant. Spin symmetric and pseudospin symmetric concepts have been used to study the aspect of deformed and superdeformation nuclei in nuclear physics. The concept of spin symmetry has been applied to the spectrum of meson and antinucleon [17], and the pseudospin symmetric concept is used to explain the quasi degeneracy of the nucleon doublets [18], super-deformation in nulei [19].

For very limited potentials, three dimensional radial Dirac equations are exactly solved only for s-wave ($1 = 0$). However, the three dimensional radial Dirac equations for the spherically symmetric potentials can only be solved approximately for $1 \neq 0$ states due to the approximation scheme of the centrifugal term ~r^(-2)[1-3, 12-16, 20].

Furthermore, the extension in higher dimensional spaces for some physical problems is very important in some physics area. The D-dimensional non-relativistic and relativistic physical systems have been investigated by many authors, such as ring-shaped pseudoharmonic potential [21], isotropic harmonic oscillator plus inverse quadratic potential [22], Pseudoharmonic potential [23], Kratzer-Fues potential [24-25],hydrogen atom [26],modified Poschl-Teller potential [27], linierly energy dependent quadratic potential [28], trigonometric scarf potential [29], ring-shaped Kratzer potential [30].

In this paper, the Dirac equation for a charged particle that moves in a field governed by q-deformed Scarf potential [15,28,31] in D dimension is investigated using finite Romanovski polynomial for exact spin symmetry case. Romanovski polynomial is traditional method which consist of reducing Schrodinger equation by an appropriate variable substitution to a form of generalized hypergeometric equation [32].

The polynomial was discovered by Sir E. J. Routh [33] and rediscovered 45 years later by V. I. Romanovski [34]. The notion "finite" refers to the observation that, for any given set of parameters only a finite of polynomials appear orthogonal [35-36]. The radial D dimensional Dirac equation reduces to one dimensional Schrodinger type equation by suitable variable change. Some of hyperbolic and trigonometric potentials are exactly solvable within the approximation of centrifugal term and their bound state solutions have been reported in the previous papers [12-16]. In the non-relativistic limit, the relativistic energy equation reduces the non-relativistic energy equation. In classical regime, the thermal properties including vibrational partition function Z, mean energy U, and specific heat C [37-38] are obtainable by applying the non-relativistic energy equation.

This paper is organized as follows. Brief review of Dirac equation in D dimension, q-deformed trigonometric potential and Romanovski polynomial are presented in section 2, solution of D dimensional Dirac equations is presented in section 3 and conclusion is presented in section 4.

Basic Theory

2.1 D dimensional Dirac Equation

The Dirac equation with the scalar potential $S(\vec{r})$ and magnitude of vector potential $V(\vec{r})$ is given as in Hu et al. [39]

$$\{\vec{\alpha}.\vec{p}+\beta(M+S(\vec{r}))\}\psi(\vec{r})=\{E-V(\vec{r})\}\psi(\vec{r}) \tag{1}$$

where *M* is the relativistic mass of the particle, *E* is the total relativistic energy, and $\vec{p}$ is the D-dimensional hyper-sphere momentum operator, $-i\nabla$

$$\vec{\alpha}=\begin{pmatrix}0 & \vec{\sigma}\\ \vec{\sigma} & 0\end{pmatrix}, \text{ and } \beta=\begin{pmatrix}I & 0\\ 0 & -I\end{pmatrix} \tag{2}$$

with $\vec{\sigma}$ are the three-dimensional Pauli matrices and I is the 2×2 identity matrix. The potential in equation (1) is spherically symmetric potential, i.e. it does not only depend on the radial coordinate $r=|\vec{r}|$, and we have taken $\hbar=1$, *c=1*.

The Dirac equation expressed in equation (18) is invariant under spatial inversion and therefore its eigen states have definite parity. By writing the spinor in D dimension as [21]

$$\psi(\vec{r})=\begin{pmatrix}\zeta(\vec{r})\\ \Omega(\vec{r})\end{pmatrix}=\frac{1}{r^{\frac{D-1}{2}}}\begin{pmatrix}F_{nK}(r)Y_{jm}^{l}(\theta,\varphi)\\ iG_{nK}(rY_{jm}^{\tilde{l}}(\theta,\varphi)\end{pmatrix} \tag{3}$$

If we insert equations (2) and (3) into equation (1) and use matrices multiplication, we achieve

$$\vec{\sigma}.\vec{p}\,\Omega(\vec{r})=\{-M-S(\vec{r})+E-V(\vec{r})\}\zeta(\vec{r}) \tag{4}$$

$$\vec{\sigma}.\vec{p}\,\zeta(\vec{r})=\{M+S(\vec{r})+E-V(\vec{r}\})\Omega(\vec{r}) \tag{5}$$

In the exact spin and p-spin symmetric cases, when the scalar potential is equal to the magnitude of vector potential $S(\vec{r})=V(\vec{r})$ and for $S(\vec{r})=-V(\vec{r})$, respectively, then the upper Dirac spinor obtained from equations (4) and (5) are

$$\vec{\sigma}.\vec{p}\,\frac{\vec{\sigma}.\vec{p}}{M+E}\zeta(\vec{r})=\{-M-2V(\vec{r})+E\}\zeta(\vec{r}) \tag{6}$$

$$\vec{\sigma}.\vec{p}\,\frac{\vec{\sigma}.\vec{p}}{M+E-2V}\zeta(\vec{r})=\{-M+E\}\zeta(\vec{r}) \tag{7}$$

By applying the Pauli matrices, it is simply shown that if $(\vec{\sigma}.\vec{p})(\vec{\sigma}.\vec{p})=p^2$, then equation (6) becomes

$$p^2+2V(\vec{r})(M+E)\zeta(\vec{r})=(E^2-M^2)\zeta(\vec{r}) \tag{8}$$

$$p^2-2V(\vec{r})(M-E)\zeta(\vec{r})=(E^2-M^2)\zeta(\vec{r}) \tag{9}$$

Since $p^2 = -\Delta_D = -\nabla_D^2$ with the hyperspherical Laplacian ∇_D^2 is given as [21]

$$\nabla_D^2 = \frac{\partial^2}{\partial r^2} + \frac{(D-1)}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\left[\frac{1}{\sin^{D-2}\theta_{D-1}}\frac{d}{d\theta_{D-1}}\left(\sin^{D-2}\theta_{D-1}\frac{d}{d\theta_{D-1}}\right) - \frac{L_{D-2}^2}{\sin^2\theta_{D-1}}\right] \tag{10}$$

then equation (8) becomes

$$\begin{aligned} &r^{1-D}\frac{\partial}{\partial r}\left\{r^{D-1}\frac{\partial}{\partial r}(\zeta(\bar{r})\right\} - \frac{1}{r^2}\left[\frac{1}{\sin^{D-2}\theta_{D-1}}\frac{d}{d\theta_{D-1}}\left(\sin^{D-2}\theta_{D-1}\frac{d(\zeta(\bar{r}))}{d\theta_{D-1}}\right) - \frac{L_{D-2}^2}{\sin^2\theta_{D-1}}(\zeta(\bar{r}))\right] \\ &-(V(r,\theta_j)(M+E))((\zeta(\bar{r})) = -(E^2 - M^2)((\zeta(\bar{r})) \end{aligned} \tag{11}$$

with the upper component of Dirac spinor is given as

$$\zeta(\bar{r}) = \zeta(\bar{x}) = \frac{1}{r^{\frac{D-1}{2}}} F_l(r) Y_{\ell_1 \ldots \ell_{D-2}.}^{\ell}(\hat{x} = \theta_1, \theta_2, \ldots, \theta_{D-1}) \tag{12}$$

$$Y_{\ell_1 \ldots \ell_{D-2}.}^{\ell}(\hat{x} = \theta_1, \theta_2, \ldots, \theta_{D-1}) = \Phi(\theta_1 = \varphi) H(\theta_2, \ldots, \theta_{D-1}) \tag{13}$$

where $\vec{x}$ is a D dimensional position vector in hyper-spherical Cartesian coordinate , the unit vector along $\vec{x}$ vector is denoted as $\hat{x} = \vec{x}/r$ and we have set $V \rightarrow (1/2)V$ in equation (11). By using variable separation method in equation (11) we have,

$$\frac{\partial^2 F_{nK}(r)}{\partial r^2} - \frac{(l_{D-1} + \frac{D-1}{2})(l_{D-1} + \frac{D-3}{2})}{r^2} F_{nK}(r) - (V(r)(M+E))F_{nK}(r) = -(E^2 - M^2)F_{nK}(r) \tag{14}$$

$$\left[\frac{1}{\sin^{j-1}\theta_j}\frac{d}{d\theta_j}\left(\sin^{j-1}\theta_j\frac{d}{d\theta_j}\right) + \ell_j(\ell_j + j - 1) - \frac{\ell_j(\ell_j + j - 2)}{\sin^2\theta_j}\right]H(\theta_j) = 0; \quad j \in [2, D-2] \tag{15}$$

$$\left[\frac{1}{\sin^{D-2}\theta_{D-1}}\frac{d}{d\theta_{D-1}}\left(\sin^{D-2}\theta_{D-1}\frac{d}{d\theta_{D-1}}\right) + \lambda_\ell - \frac{1}{\sin^2\theta_{D-2}}(L_{D-2}^2) - (E+M)V(\theta_{D-1})\right]H(\theta_{D-1}) = 0 \tag{16}$$

The D dimensional relativistic wave function are obtained from equations (14-16) with D is the dimension of the space.

2.2 **Review of q-deformed trigonometric function**

The *q*-deformed trigonometric function is formulated in the same way with the formulation of q-deformed hyperbolic function introduced by Arai [40] some years ago, in accordingly the q-deformed trigonometric function is defined as the definition of trigonometric function as follows:

$$\sin_q ar = \frac{e^{iar} - qe^{-iar}}{2} \quad \cos_q ar = \frac{e^{iar} + qe^{-iar}}{2} \quad \sin_q^2 ar + \cos_q^2 ar = q \tag{18}$$

$$\tan_q ar = \frac{\sin_q ar}{\cos_q ar} \quad \sec_q ar = \frac{1}{\cos_q ar} \quad 1 + \tan_q^2 ar = q\sec_q^2 ar \tag{19}$$

$$\frac{d\sin_q ar}{dr} = a\cos_q ar \; ; \; \frac{d\tan_q ar}{dr} = qa\sec_q^2 ar \tag{20}$$

By a convenient translation of the spatial variable, one can transform the deformed potentials into the corresponding non-deformed ones or vice-versa. In analogy to the translation of spatial variable for hyperbolic function introduced by Dutra [41] we propose the translation of spatial variable for trigonometric function as follows

$$r \to r + \frac{\ln\sqrt{q}}{i\alpha}, \text{and} \;\; r \to r - \frac{\ln\sqrt{q}}{i\alpha} \tag{21}$$

and then by inserting equation (22) into equations (19) and (20) we have

$$\sin_q \alpha r \to \sqrt{q}\sin\alpha r; \quad \cos_q \alpha r \to \sqrt{q}\cos\alpha r; \text{ or } \sin\alpha r \to \frac{\sin_q \alpha r}{\sqrt{q}}; \quad \cos\alpha r \to \frac{\cos_q \alpha r}{\sqrt{q}}; \tag{22}$$

The translation of spatial variable in equation (21) can be used to map the energy and wave function of non-deformed potential toward deformed potential of Scarf potential as in the case of hyperbolic Scarf potential [12, 15, 42].

2.3 Method of Analysis

The Romanovski polynomials is used to solve the Dirac equation in the limit of spin symmetric and pseudospin symmetric cases since in the spin and pseudospin symmetric limited, the Dirac equations reduce to one dimensional Schrodinger-like equation. One dimensional Schrodinger equation of potential of interest reduces to the differential equation of Romanovski polynomial by appropriate variable and wave function substitutions. The one dimensional Schrodinger equation is given as

$$-\frac{\hbar^2}{2M}\frac{\partial^2\Psi(x)}{\partial x^2} + V(x)\Psi(x) = E\Psi(x) \tag{23}$$

where V(x) is an effective potential which is mostly shape invariant potential. By suitable variable substitution $x = f(s)$ equation (23) changes into generalized hypergeometric type equation expressed as

$$\frac{\partial^2\Psi(s)}{\partial s^2} + \frac{\tilde{\tau}(s)}{\sigma(s)}\frac{\partial\Psi(s)}{\partial s} + \frac{\tilde{\sigma}(s)}{\sigma^2(s)}\Psi(s) = 0 \tag{24}$$

with $\sigma(s)$ and $\tilde{\sigma}(s)$ are mostly polynomials of order two , $\tilde{\tau}(s)$ is polynomial of order one, s, $\sigma(s)$, $\tilde{\sigma}(s)$, and $\tilde{\tau}(s)$ can have any real or complex values [47]. Equation (24) is solved by variable separation method. By introducing new wave function in equation (24)

$$\Psi_n(r) = g_n(s) = (1+s^2)^{\frac{\beta}{2}} e^{\frac{-\alpha}{2}\tan^{-1}s} D_n^{(\beta,\alpha)}(s) \tag{25}$$

we obtain a hypergeometric type differential equation, which can be solved using finite Romanovski polynomials [12-16, 43-44] is expressed as

$$\sigma(s)y''(s) + \tau(s)y'(s) + \lambda y(s) = 0 \tag{26}$$

$$\text{with } \sigma(s) = as^2 + bs + c; \;\; \tau = fs + h \text{ and } -\{n(n-1) + 2n(1-p)\} = \lambda = \lambda_n \tag{27}$$

$$\text{and } y_n = R_n^{(p,q')}(s) = D_n^{(\beta,\alpha)}(s) \tag{28}$$

For Romanovski polynomials, the values of parameters in equation (27) are

$a = 1$, $b = 0$, $c = 1$, $f = 2(1\text{-}p)$ and $h = q'$ with $p > 0$ (29)

therefore equation (26) is rewritten as

$$(1+s^2)\frac{\partial^2 R_n^{(p,q')}}{\partial s^2} + \{2s(-p+1) + q'\}\frac{\partial R_n^{(p,q')}(s)}{\partial s} - \{n(n-1) + 2n(1-p)\}R_n^{(p,q')}(s) = 0 \tag{30}$$

Equation (26) is described in the textbook by Nikiforov-Uvarov [44] where it is cast into self adjoint form and its weight function, $w(s)$, satisfies Pearson differential equation

$$\frac{d(\sigma(s)w(s))}{ds} = \tau(s)w(s) \tag{31}$$

The weight function, $w(s)$, is obtained by solving the Pearson differential equation expressed in equation (32) and by applying condition in equations (27) and (29), given as

$$w^{(p,q')}(s) = (1+s^2)^{-p} e^{q'\tan^{-1}(s)} \quad (32)$$

The corresponding polynomials are classified according to the weight function, and are built up from the Rodrigues representation which is presented as

$$y_n = \frac{B_n}{w(s)} \frac{d^n}{ds^n} \left\{ \left(as^2 + bs + c \right)^n w(s) \right\} \quad (33)$$

and is given as

$$R_n^{(p,q')}(s) = D_n^{(\beta,\alpha)}(s) = \frac{1}{(1+s^2)^{-p} e^{q'\tan^{-1}(s)}} \frac{d^n}{ds^n} \left\{ (1+s^2)^n (1+s^2)^{-p} e^{q'\tan^{-1}(s)} \right\} \quad (34)$$

for Romanovski polynomial with B_n is a normalization constant, and for $\sigma(s) > 0$ and $w(s)>0$, $y_n(s)$'s are normalized polynomials and are orthogonal with respect to the weight function $w(s)$ within a given interval (s_1, s_2), which is expressed as

$$\int_{-\infty}^{\infty} w(s) y_n(s) y_{n'}(s) ds = \delta_{nn'} \quad (35)$$

This weight function in equation (31) first reported by Routh [33] and then by Romanovski [34]. The polynomial associated with equation (34) are named after Romanovski and will be denoted by $R_n^{(p,q)}(s)$. Due to the decrease of the weight function by s^{-2p}, integral of the type

$$\int_{-\infty}^{\infty} w^{(p,q')} R_n^{(p,q')}(s) R_{n'}^{(p,q')}(s) ds \quad (36)$$

will be convergent only if $n' + n < 2p - 1$ (37)

This means that only a finite number of Romanovski polynomials are orthogonal, and the orthogonality integral of the polynomial is expressed similar to the equation (35) where $y_n = R_n^{(p,q')}(s)$.

The orthogonality integral of the wave functions expressed in equation (39) gives rise to orthogonality integral of the finite Romanovski polynomials, that is given as

$$\int_0^{\infty} \Psi_n(r) \Psi_{n'}(r) dr = \int_{-\infty}^{\infty} w^{(p,q')} R_n^{(p,q')}(s) R_{n'}^{(p,q')}(s) ds \quad (38)$$

In this case the values of p and q' are not n-dependence where n is the degree of polynomials. However, if either equation (35) or (36) is not fulfilled then the Romanovski polynomials is infinity [43].

3. Solution of Dirac Equation for central Potential in D dimension

In order to solve the radial Dirac equation in equation (14), we use the approximation value for the centrifugal term as in Greene and Aldirch, and in Ikdhair [20-21], $\frac{1}{r^2} \cong a^2 \left(d_0 + \frac{1}{\sin_q^2 ar} \right)$, for $tr << 1$ and $d_0 = 1/12$

For radial q-deformed trigonometric Scarf potential which is given as

$$V(r, \theta_j) = V(r) = a^2 \left\{ \frac{V_0}{\sin_q^2 ar} - \frac{V_1 \cos_q ar}{\sin_q^2 ar} \right\} \quad (39)$$

the D dimensional Dirac equation in equation (14) becomes

$$\frac{d^2F(r)}{dr^2}-a^2\left(\frac{V_0(E+M)+(\tilde{l}+\frac{D-1}{2})(\tilde{l}+\frac{D-3}{2})}{\sin_q^2 ar}-\frac{(E+M)V_1\cos_q ar}{\sin_q^2 ar}\right)F(r) \tag{40}$$

$$+(E^2-M^2-a^2(\tilde{l}+\frac{D-1}{2})(\tilde{l}+\frac{D-3}{2})d_0)F(r)=0$$

By setting

$$V_0(E+M)+(\tilde{l}+\frac{D-1}{2})(\tilde{l}+\frac{D-3}{2})=V_0' \tag{41}$$

$$V_1(E+M)=V_1',\ E^2-M^2-a^2(\tilde{l}+\frac{D-1}{2})(\tilde{l}+\frac{D-3}{2})d_0=a^2E' \tag{42}$$

equation (40) becomes one dimensional Schrodinger type equation

$$\frac{d^2F(r)}{dr^2}-a^2\left(\frac{V_0'}{\sin_q^2 ar}-\frac{V'_1\cos_q ar}{\sin_q^2 ar}\right)F(r)=-a^2E'F(r) \tag{43}$$

By variable substitution given as $\cos_q ar = ix\sqrt{q}$ in equation (43) we obtain

$$\left\{(1+x^2)\frac{\partial^2}{\partial x^2}+x\frac{\partial}{\partial x}+\frac{V'_0}{q(1+x^2)}-\frac{V'_1 i\sqrt{q}x}{q(1+x^2)}-E'\right\}F_{n\kappa}(x)=0 \tag{44}$$

By applying equation (25) into equation (44) we have

$$(1+x^2)\frac{\partial^2 D_n^{(\beta,\alpha)}(x)}{\partial x^2}+\{x(2\beta+1)-\alpha\}\frac{\partial D_n^{(\beta,\alpha)}(x)}{\partial x}-\left\{\frac{q\beta\alpha x-\frac{q}{2}\alpha x-\frac{q\alpha^2}{4}+q\beta^2-q\beta-V'_0+V'_1 ix\sqrt{q}}{q(1+x^2)}+E_s'-\beta^2\right\}D_n^{(\beta,\alpha)}(x)=0 \tag{46}$$

By setting the numerator of the term that has denominator of $q(1+x^2)$ to be zero then we get

$$q\beta\alpha-\frac{q}{2}\alpha+V'_1 i\sqrt{q}=0 \quad \alpha=-\frac{V'_1 i\sqrt{q}}{q(\beta-1/2)}=-\frac{V'_1 i}{\sqrt{q}(\beta-1/2)} \tag{47}$$

and equation (46) reduces to

$$(1+x^2)\frac{\partial^2 D_n^{(\beta,\alpha)}(x)}{\partial x^2}+\{x(2\beta+1)-\alpha\}\frac{\partial D_n^{(\beta,\alpha)}(x)}{\partial x}-\{E_s'-\beta^2\}D_n^{(\beta,\alpha)}(x)=0 \tag{48}$$

From the two equations in equation (47) we get

$$(\beta-1/2)=\frac{1}{2}\left\{\sqrt{\frac{(q/4+V'_0+V'_1\sqrt{q})}{q}}-\sqrt{\frac{(q/4+V'_0-V'_1\sqrt{q})}{q}}\right\} \tag{49}$$

$$\alpha=-\frac{V'_1 i}{\sqrt{q}(\beta-1/2)}=-i\left\{\sqrt{\frac{(q/4+V'_0+V'_1\sqrt{q})}{q}}+\sqrt{\left(\frac{q/4+V'_0-V'_1\sqrt{q}}{q}\right)}\right\} \tag{50}$$

By comparing equations (30) and (48) we get

$$(2\beta+1)=2(1-p)\ ;\ \alpha=-q'\ ;\ E'-\beta^2=n(n-1)+2n(1-p) \tag{52}$$

and from those three equations in equation (52) we get

$$E' = (n+\beta)^2 \tag{53}$$

$$q' = i\left\{\sqrt{\frac{\left(q/4+V'_0+V'_1\sqrt{q}\right)}{q}} + \sqrt{\left(\frac{q/4+V'_0-V'_1\sqrt{q}}{q}\right)}\right\} \tag{54}$$

$$p = -\frac{1}{2}\left\{\sqrt{\frac{\left(q/4+V'_0+V'_1\sqrt{q}\right)}{q}} - \sqrt{\frac{\left(q/4+V'_0-V'_1\sqrt{q}\right)}{q}}\right\} \tag{55}$$

The relativistic energy equation which is obtained from equations (42), (49), (53) is given as

$$E^2 - M^2 = a^2(\tilde{l} + \frac{D-1}{2})(\tilde{l} + \frac{D-3}{2})d_0 + a^2\left\{n + 1/2 + \left\{\sqrt{\frac{\left(q/4+V'_0+V'_1\sqrt{q}\right)}{4q}} - \sqrt{\frac{\left(q/4+V'_0-V'_1\sqrt{q}\right)}{4q}}\right\}\right\}^2 \tag{56}$$

By using equations (25), (28), (34), (49), (50), and (52) we get the the relativistic wave functions for Dirac spinor upper components. The ground state and first excited state of Dirac spinnor upper component are given as

$$F_{0\kappa}(s) = \left(\frac{\sin_q^2 ar}{q}\right)^{V_+/2}\left(1 - \frac{\cos_q ar}{\sqrt{q}}\right)^{-\{V_+ + V_-\}/2} \tag{57}$$

$$F_{1\kappa}(s) = -\frac{i}{a}\left\{(1+V_+)2\cot_q ar - \frac{(V_+ + V_-)}{\sqrt{q} - \cos_q ar}\left(a\sin_q ar\right)\right\}\left(\frac{\sin_q^2 ar}{q}\right)^{(V_+ + 1)/2}\left(1 - \frac{\cos_q ar}{\sqrt{q}}\right)^{-\{V_+ + V_-\}/2} \tag{58}$$

The higher levels of excited state of of Dirac spinors are expressed in term of Romanovski polynomials that are obtained using equations (25), (28), (34), (49), (50), and (52).

Thermodynamical Properties of Material

In non-relativistic condition, the relativistic energy equation expressed in equation (56) together with equations (41-42) reduces into non-relativstic energy by taking $(M+E) \to 2\mu$ where μ is the non-relativistic mass, $(E-M) \to E_{NR}$, E_{NR} is the non-relativistic energy, then $E^2 - M^2 = (E+M)(E-M) = 2\mu E_{NR}$ and equation (56) becomes

$$E_{NR} = \frac{a^2}{2\mu}\left\{(\tilde{l} + \frac{D-1}{2})(\tilde{l} + \frac{D-3}{2})d_0 + \left[(n + 1/2 + \left\{\sqrt{\frac{\left(q/4+V'_{0NR}+V'_{1NR}\sqrt{q}\right)}{4q}} - \sqrt{\frac{\left(q/4+V'_{0NR}-V'_{1NR}\sqrt{q}\right)}{4q}}\right\}\right]^2\right\} \tag{59}$$

with $V_0 2\mu + (\tilde{l} + \frac{D-1}{2})(\tilde{l} + \frac{D-3}{2}) = V_{0NR}'$ and $V_1 2\mu = V_{1NR}'$ (60)

Since d_0 is small then equation (45) reduces to

$$E_{NR} = \frac{a^2}{2\mu}\left[(n + 1/2 + \left\{\sqrt{\frac{\left(q/4+V'_{0NR}+V'_{1NR}\sqrt{q}\right)}{4q}} - \sqrt{\frac{\left(q/4+V'_{0NR}-V'_{1NR}\sqrt{q}\right)}{4q}}\right\}\right]^2 = \frac{a^2}{2\mu}(\xi - n)^2 \tag{61}$$

with $\xi = -\left[1/2 + \left\{\sqrt{\frac{\left(q/4 + V'_{0NR} + V'_{1NR}\sqrt{q}\right)}{4q}} - \sqrt{\frac{\left(q/4 + V'_{0NR} - V'_{1NR}\sqrt{q}\right)}{4q}}\right\}\right]$ (62)

In classical regimes [42], the vibrational partition function, vibrational mean energy, and specific heat are obtained from the non-relativistic energy equation in Eq. (61). The vibrational partition function is defined as

$$Z(\zeta,\beta) = \sum_{n=0}^{\zeta} e^{-\beta E_{NR}}, \beta = \frac{1}{kT} \tag{63}$$

k is Boltzman constant, E_{nl} is non-relativistic energy spectrum of the system. By setting $\frac{1}{\delta^2} = \frac{a^2\beta}{2\mu}$ and in the classical regime when the temperature, *T*, is high enough, then the value of ξ is high, and β is small therefore equation (63) could be written into integral form as

$$Z(\zeta,\beta) = \sum_{n=0}^{\frac{\xi}{\delta}} e^{-\left\{\frac{n-\xi}{\delta}\right\}^2} = \sum_{n=0}^{\frac{\xi}{\delta}} e^{-y^2} = \delta\int^{\frac{\xi}{\delta}} e^{-y^2}\,dy = \delta\frac{\sqrt{\pi}}{2} erf\left(\frac{\xi}{\delta}\right), \tag{64}$$

with $y = \frac{n-\xi}{\delta}$ and erf is the error function given as [45]

The vibrational specific heat and the vibrational mean energy are defined as

$$U(\beta,\xi) = -\frac{\partial}{\partial\beta}\ln Z(\xi,\beta) \quad \text{and} \quad C = -\frac{\partial}{\partial T}U = -k\beta^2\frac{\partial}{\partial\beta}U \tag{65}$$

By using equations (64) and (65) one can obtain the partition function, vibrational specific heat and the vibrational mean energy numerically using Mat Lab.

Conclusion

The D dimensional Dirac equation for q-deformed trigonometric Scarf potential in the scheme of centrifugal term approximation is exactly solved using Romanovski polynomials in the case of exact spin symmetric case. The D dimensional relativistic energy spectra are obtainable numerically from D dimensional relativistic energy equation using Matlab are always positive for spin symmetric case. In the non-relativistic limit, the non-relativistic spectra is denerated from the relativistic energy equation by suitable parameter substitution. In the classical limit, the thermodynamics properties are obtainable from non-relativistic energy equation. The upper component of Dirac spinors are obtained in the form of Romanovski polynomial. Acknowledgement

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