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ANALYTICAL SOLUTION OF THE DIRAC EQUATION FOR TRIGONOMETRIC SCARF II POTENTIAL PLUS TRIGONOMETRIC POSCH-TELLER NON-CENTRAL POTENTIAL USING ASYMPTOTIC ITERATION METHOD

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ABSTRACT

Analytical solutions of the Dirac equation for trigonometric Scarf II potential plus trigonometric Posch-Teller non-central potential for spin symmetric case was solved by using asymptotic iteration method. The three-dimensional Dirac equation was separated to radial part and angular part. The radial part and angular part of Dirac equation were reduced to hypergeometry-type equation and then we transform it to AIM-type differential equation to obtain the analytical solution. We obtained relativistic energy equation and the radial wavefunctions from the radial part solution. Then the angular wavefunctions and the orbital quantum number equation were obtained from angular part solution.

Key Word: the Dirac Equation for spin symmetric, Trigonometric Scarf II Potential, Trigonometric Posch-Teller Non-central Potential, Asymptotic Iteration Method

INTRODUCTION

The Dirac equation is the relativistic wave equation which was formulated by Paul Dirac on 1928. In quantum mechanics, the Dirac equation always describes particles dynamic with spin-1/2 (Greiner, 2000). Particles that move in the field of potential require relativistic effect. Relativistic influence can be formulated with the Klein-Gordon equations or the Dirac equation. Potential fields can affect the behavior of particles that relies on potential field.

The solution of Dirac equation for some typical potentials under special cases of spin symmetry and pseudospin symmetry have been investigated (Hamzavi, 2013). For symmetric spin case, $\kappa(\kappa+1) = l(l+1)$ that gives $\kappa = l = j + 1/2$ for $\kappa > 0$ and $\kappa = -(l+1) = -(j+1/2)$ for $\kappa < 0$ where $\kappa = (\vec{\sigma} \cdot \vec{L} + 1)$ with κ is the eigenvalues of the spin orbit coupling operator, l is the angular momentum quantum number and j is the total of angular momentum quantum number. For pseudospin symmetric case, $\kappa(\kappa-1) = \tilde{l}(\tilde{l}+1)$ that gives $\kappa = \tilde{l} + 1 = j + 1/2$ for $\kappa > 0$ and $\kappa = -\tilde{l} = -(j+1/2)$ for $\kappa < 0$ (Suparmi, 2014). The concept of spin symmetry has been

applied to the spectrum of meson and antinucleon, and the pseudospin symmetric concept is used to explain the quasi degeneracy of the nucleon doublets, exotic nuclei, super-deformation in nuclei, and to establish an affective nuclear shell-model scheme (Suparmi, 2014). The spin symmetry occurs when the difference between vector potential and scalar potential is zero, while the pseudospin symmetry arises when the sum of vector and scalar potentials are constant (Suparmi, 2014).

Some researchers have solved the Dirac equation with various potentials, it included the Eckart potential (Bahar, 2013), 3D harmonics oscillator plus trigonometric scarf non-central potential (Cari, 2014), Scarf Potential with the Poschl-Teller Non-central potential (Cari, 2012), the deformed Woods-Saxon potential (Falaye et al, 2012), Pöschl-Teller Potential including Tensor Coupling (Hamzavi, 2013), Hulthen plus trigonometric Rosen Morse Non-Central Potential (Suparmi et al, 2014), q-Deformed Trigonometric Scarf potential with q-Deformed Trigonometric Tensor Coupling Potential (Suparmi et al, 2014) and others. The various methods have been used, it included hypergeometric method, SUSY quantum mechanics (Cari & Suparmi, 2014), Romanovski polynomials (Suparmi et

al, 2014), the Nikiforov-Uvarov method (Deta et al, 2013), asymptotic iteration method (Soylu et al, 2008) and others.

In this paper, the relativistic energy and wavefunctions of trigonometric Scarf II potential with trigonometric Posch-Teller, which non central potential, are analyzed using iteration asymptotic method. This paper is organized as follows asymptotic iteration method is presented in section 2. Solution of radial and angular Dirac equations are presented in section 3.1 and 3.2. Conclusion is presented in section 4.

ASYMPTOTIC ITERATION METHOD

Asymptotic Iteration Method (AIM) is used to solve second order differential equation in terms (Falaye dkk, 2012):

$$y_n''(x) - \lambda_0(x)y_n'(x) - s_0(x)y_n(x) = 0 \tag{1}$$

where $\lambda_0(x) \neq 0$ and $s_0(x)$ are coefficients of the differential equation and they are well defined functions as well as sufficiently differentiable.

The solution of equation (1) can be obtained by using iteration of λ_i and S_i ,

$$\begin{aligned} \lambda_i(x) &= \lambda'_{i-1} + \lambda_{i-1}\lambda_0 + s_{i-1} \\ s_i(x) &= s'_{i-1} + s_0\lambda_{i-1} \\ i &= 1, 2, 3, \dots \end{aligned} \tag{2}$$

Eigenvalues from equation (1) can be obtained using equation (3):

$$\lambda_i(x)s_{i-1}(x) - \lambda_{i-1}(x)s_i(x) = 0 = \Delta_i, i = 1, 2, 3, \dots \tag{3}$$

On the other hand, equation (1) can be written in term:

$$y_n'' = 2 \left(\frac{tx^{N+1}}{1-bx^{N+2}} - \frac{c+1}{x} \right) y_n'(x) - \frac{Wx^N}{1-bx^{N+2}} \tag{4}$$

Equation (4) is AIM-type differential equation which is solved by using equation (5), (Soylu et al, 2008;Falaye et al, 2012)

$$y_n = (-1)^n C(N+2)^n (\sigma)_{n2} F_1(-n, p+n, \sigma, bx^{N+2}) \tag{5}$$

where,

$$(\sigma)_n = \frac{\Gamma(\sigma+n)}{\Gamma(\sigma)}, \sigma = \frac{2c+N+3}{N+2}, p = \frac{(2c+1)b+2t}{(N+2)b} \tag{6}$$

Here C' is normalization constant and ${}_2F_1$ is hypergeometric function. Equation (5) is eigenfunctions of AIM-type differential equation in equation (4). Equation (5) is used to obtain wavefunctions of the Dirac equation. The relativistic energy equation can be

formulated by equating eigenvalue equation by using equation (3).

DIRAC EQUATION

The Dirac equation for fermionic massive spin-1/2 particles subject to vector and scalar potentials (Suparmi et al, 2014). The Dirac equation with the attractive scalar potential $S(\vec{r})$ and the magnitude of the repulsive vector potential $V(\vec{r})$ by setting $\hbar=c=1$ is given as

$$\{c\vec{\alpha} \cdot \vec{p} + \beta(Mc^2 + S(\vec{r}))\}\psi(\vec{r}) = \{E - V(\vec{r})\}\psi(\vec{r}) \tag{7}$$

where M is relativistic mass of particle, E is total energy, and \mathbf{p} is the linier momentum operator, value of α and β is given as

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \text{ and } \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \tag{8}$$

with I is 2 x 2 identity matrix and $\vec{\sigma}$ is three dimensional Pauli matrices

$$\vec{\sigma}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \vec{\sigma}_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \vec{\sigma}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{9}$$

In the Pauli-Dirac representation, let

$$\psi(\vec{r}) = \begin{pmatrix} f_{nk}(\vec{r}) \\ g_{nk}(\vec{r}) \end{pmatrix} \tag{10}$$

We have

$$c\vec{\sigma} \cdot \vec{p} g_{nk}(\vec{r}) = [E - V(\vec{r}) - \vec{M} - S(r)] f_{nk}(\vec{r}) \tag{11a}$$

$$c\vec{\sigma} \cdot \vec{p} f_{nk}(\vec{r}) = [E - V(\vec{r}) + \vec{M} + S(r)] g_{nk}(\vec{r}) \tag{11b}$$

For spin symmetry, equation (11b) becomes

$$c\vec{\sigma} \cdot \vec{p} f_{nk}(\vec{r}) = [E - (V(\vec{r}) - S(r))] M g_{nk}(\vec{r})$$

$$g_{nk}(\vec{r}) = \frac{\vec{\sigma} \cdot \vec{p} f_{nk}(\vec{r})}{[E - (V(\vec{r}) - S(r))] M} \tag{12}$$

Substituting equation (12) into (11a), we have

$$p^2 f_{nk}(\vec{r}) = \{E + M - C_s\} \{[E - M - \Sigma(r)]\} f_{nk}(\vec{r}) \tag{13}$$

where $p^2 = \nabla$ then equation (24) becomes

$$\nabla f_{nk}(\vec{r}) + \{E + M - C_s\} \{[E - M - \Sigma(r)]\} f_{nk}(\vec{r}) = 0 \tag{14}$$

In spherical coordinates, Trigonometri Scarf II potential combined with trigonometric posch-teller potential is defined as

$$V(r, \theta) = \frac{V_0}{\sin^2 \alpha r} - \frac{V_1 \cos \alpha r}{\sin^2 \alpha r} + \frac{1}{r^2} \left(\frac{\kappa(\kappa-1)}{\sin^2 \theta} + \frac{\eta(\eta-1)}{\cos^2 \theta} \right) \quad (15)$$

Putting equation (15) into equation (14) and simplifying the equation, and let

$$f_{nk}(\vec{r}) = f_{nk}(r, \theta, \varphi) = R(r)P(\theta)\Phi(\varphi) \quad (16)$$

$$\left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \theta} \left(\sin^2 \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} \right) R(r)P(\theta)\Phi(\varphi) -$$

$$\left(\alpha^2 \left(\frac{V_0}{\sin^2 \alpha r} - \frac{V_1 \cos \alpha r}{\sin^2 \alpha r} \right) - \frac{1}{r^2} \left(\frac{\kappa(\kappa-1)}{\sin^2 \theta} + \frac{\eta(\eta-1)}{\cos^2 \theta} \right) \right) (E + M - C_s) R(r)$$

$$P(\theta)\Phi(\varphi) + (E - M)(E + M - C_s)R(r)P(\theta)\Phi(\varphi) = 0 \quad (17)$$

Separating the variables in equation (17), we obtain

$$\frac{1}{R} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) - (E + M - C_s) r^2 \alpha^2 \left(\frac{V_0}{\sin^2 \alpha r} - \frac{V_1 \cos \alpha r}{\sin^2 \alpha r} \right) + \quad (18a)$$

$$(E - M)(E + M - C_s) r^2 = l(l + 1) - \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) - \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} + (E + M - C_s) \quad (18b)$$

$$\left(\frac{\kappa(\kappa-1)}{\sin^2 \theta} + \frac{\eta(\eta-1)}{\cos^2 \theta} \right) = l(l + 1) - \frac{1}{\phi(\varphi)} \frac{\partial^2 \phi(\varphi)}{\partial \varphi^2} = m^2 \quad (18c)$$

Equation (28c) is well known with this solution

$$\phi_m = \frac{1}{\sqrt{2\pi}} e^{im\varphi}, m = 0, \pm 1, \pm 2, \dots \quad (19)$$

SOLUTION OF RADIAL DIRAC EQUATION FOR NON-CENTRAL POTENTIAL

$$\left(\frac{d^2}{dr^2} - \frac{k(k+1)}{r^2} - (E_{nk} + M - C) \right) \left(\frac{E_{nk} - M + \frac{\alpha^2 V_0}{\sin^2 \alpha r} + \frac{\alpha^2 V_1 \cos \alpha r}{\sin^2 \alpha r} \right) G_{nk}(r) = 0 \quad (20)$$

with $\frac{1}{r^2} \equiv \frac{\alpha^2}{\sin^2 \alpha r}$

By inserting value $\frac{1}{r^2}$ then equation (20), become

$$\left(\frac{d^2}{dr^2} - \frac{\alpha^2}{\sin^2 \alpha r} (k(k+1) + V_0(E_{nk} + M - C)) + \frac{\alpha^2 V_1 \cos \alpha r}{\sin^2 \alpha r} \right) F_{nk}(r) = 0 \quad (21)$$

by using

$$A_s = k(k+1) + V_0(E_{nk} + M - C) \quad (22a)$$

$$B_s = V_1(E_{nk} + M - C) \quad (22b)$$

$$E'_s = \frac{(E_{nk} + M - C)(E_{nk} - M)}{\alpha^2} \quad (23c)$$

then the equation (35), become

$$\left\{ \frac{d^2}{dr^2} - \frac{\alpha^2 A_s}{\sin^2 \alpha r} + \frac{\alpha^2 B_s \cos \alpha r}{\sin^2 \alpha r} - \alpha^2 E'_s \right\} F_{nk}(r) = 0 \quad (23)$$

By variabel substitution $\cos(\alpha r) = 1 - 2z$ into equation (23), So obtained

$$z(1-z) \frac{\partial^2 F_{nk}(z)}{\partial z^2} + \left(\frac{1}{2} - z \right) \frac{\partial F_{nk}(z)}{\partial z} + \left\{ E'_s - \frac{A_s - B_s}{4z} - \frac{A_s + B_s}{4(1-z)} \right\} F_{nk}(z) = 0 \quad (24)$$

By using

$$F_{nk}(z) = (z)^\delta (1-z)^\gamma f_n(z) \quad (25)$$

and simplifying it, equation (25) can be transformed to hypergeometric differential equation

$$z(1-z) f_n''(z) + \left(2\delta + \frac{1}{2} - (2\delta + 2\gamma + 1)z \right) f_n'(z) + \left((\delta + \gamma)^2 - E'_s \right) f_n(z) = 0 \quad (26)$$

Equation (26) is hypergeometri differential equation, so we must transform it to AIM-type equation by divide equation (26) with $(z(1-z))$, yields

$$f_n''(z) = \left\{ \frac{- \left(2\delta + \frac{1}{2} - (2\delta + 2\gamma + 1)z \right)}{z(1-z)} f_n'(z) \right\} + \left\{ \frac{(\delta + \gamma)^2 - E'_s}{z(1-z)} f_n(z) \right\} \quad (27)$$

From equation (27) obtained

$$\lambda_0 = \frac{-\left(2\delta + \frac{1}{2}\right) + (2\delta + 2\gamma + 1)z}{z(1-z)} \quad (28a)$$

$$s_0 = \left\{ \frac{(\delta + \gamma)^2 - E'}{z(1-z)} \right\} \quad (28b)$$

Then, will be done iteration λ_i dan s_i using equation (2), where i is iteration. And to obtain eigenvalue of energy, we using the equation (3). And by using software matlab, we obtained the results as follows:

$$\begin{aligned} \Delta_0 \varepsilon_0 &= (\delta + \gamma)^2 \\ \Delta_1 \varepsilon_1 &= (2\delta + 2\gamma + 1) + (\delta + \gamma)^2 = (\delta + \gamma + 1)^2 \\ \Delta_2 \varepsilon_2 &= (4\delta + 4\gamma + 4) + (\delta + \gamma)^2 = (\delta + \gamma + 2)^2 \end{aligned} \quad (29)$$

$$\Delta_3 \varepsilon_3 = (6\delta + 6\gamma + 9) + (\delta + \gamma)^2 = (\delta + \gamma + 3)^2$$

$\Delta_i \dots\dots$

where $\varepsilon = E'$. From equation (29), can be generalized become:

$$\varepsilon_{n_r} = (\delta + \gamma + n_r)^2 \quad (30)$$

where n_r is radial quantum number ($n_r = 1, 2, 3, \dots$)

$$\delta = \frac{1}{2} \left\{ \frac{1}{2} - \sqrt{A_s - B_s + \frac{1}{4}} \right\} \quad (31a)$$

$$\gamma = \frac{1}{2} \left\{ \frac{1}{2} + \sqrt{A_s + B_s + \frac{1}{4}} \right\} \quad (31b)$$

Substituting equation (31a) and equation (31b) into equation (30), then we obtained the relativistic energy,

$$E'_s = \left(\frac{1}{2} \sqrt{A_s + B_s + \frac{1}{4}} - \frac{1}{2} \sqrt{A_s - B_s + \frac{1}{4}} + n_r + \frac{1}{2} \right)^2 \quad (32)$$

The wavefunction of the system are obtained by comparing between equation (26) with equation (4), so we have

$$\begin{aligned} c &= \frac{2\delta - \frac{3}{2}}{2}, N = -1, t = \frac{2\gamma + \frac{1}{2}}{2}, b = 1 \\ \sigma &= \frac{2c + N + 3}{N + 2} = 2\delta + \frac{1}{2}, p = \frac{(2c + 1)2t}{(N + 2)b} = 2\delta + 2\gamma \end{aligned} \quad (33)$$

From equation (5), we have

$$f_{n_r}(z) = (-1)^{n_r} C'(1)^{n_r} \left(2\delta + \frac{1}{2} \right)_{n_r, 2} F_1 \left(-n_r, 2\delta + 2\gamma + n_r, 2\delta + \frac{1}{2}, z \right) \quad (34)$$

By substituting Equation (34) into Equation (25), we have the radial wave function,

$$F_{nk} = (z)^\delta (1-z)^\gamma (-1)^{n_r} C'(1)^{n_r} \left(2\delta + \frac{1}{2} \right)_{n_r, 2} F_1 \left(-n_r, 2\delta + 2\gamma + n_r, 2\delta + \frac{1}{2}, z \right) \quad (35)$$

Where $z = \frac{1 - \cos(\alpha r)}{2}$

$$F_{nk} = \left(\frac{1 - \cos(\alpha r)}{2} \right)^\delta \left(\frac{1 + \cos(\alpha r)}{2} \right)^\gamma (-1)^{n_r} C'(1)^{n_r} \left(2\delta + \frac{1}{2} \right)_{n_r, 2} F_1 \left(-n_r, 2\delta + 2\gamma + n_r, 2\delta + \frac{1}{2}, z \right) \quad (36)$$

Where C_{nr} is radial normalization constant, ${}_2F_1$ is

hypergeometric function and $\left(2\delta + \frac{1}{2} \right)_{n_r}$ is Pochamer symbol.

SOLUTION OF POLAR DIRAC EQUATION

$$\begin{aligned} \frac{d}{d\theta} \left(\sin \theta \frac{d\Theta(\theta)}{d\theta} \right) - (E + M - C) \sin^2 \theta \left(\frac{\kappa(\kappa - 1)}{\sin^2 \theta} + \frac{\eta(\eta - 1)}{\cos^2 \theta} \right) \Theta(\theta) + \\ l(l + 1) \sin^2 \theta \Theta(\theta) - m^2 \frac{\Theta(\theta)}{\sin \theta} = 0 \end{aligned} \quad (37)$$

By setting $P = H / \sqrt{\sin \theta}$ in equation (37)

$$\begin{aligned} \frac{d^2 H}{d\theta^2} + \frac{1}{2} H + \frac{1}{4} \frac{\cos^2 \theta}{\sin^2 \theta} H - (E + M - C) \left(\frac{\kappa(\kappa - 1)}{\sin^2 \theta} + \frac{\eta(\eta - 1)}{\cos^2 \theta} \right) H + l(l + 1) H - \frac{m^2}{\sin^2 \theta} H = 0 \end{aligned} \quad (38)$$

To solve equation (37) we introduce a new variable $\cos^2 \theta = z$ and equation (37) becomes

$$z(1-z)\frac{d^2H}{dz^2} + \left(\frac{1}{2}-z\right)\frac{dH}{dz} - \left\{ (E+M-C)\left(\frac{\kappa(\kappa-1)}{4(1-z)} + \frac{\eta(\eta-1)}{4z}\right) + \frac{m^2 - 1/4}{4(1-z)} \right\} H + \left(\frac{l + \frac{1}{2}}{4}\right) H = 0 \quad (39)$$

By using

$$H = z^\alpha (1-z)^\beta p \quad (40)$$

and simplifying it, equation (39) can be transformed to hypergeometric differential equation:

$$z(1-z)p'' + p' \left(\left(2\alpha + \frac{1}{2}\right) - (2\alpha + 2\beta + 1)z \right) + p \left(-(\alpha + \beta)^2 + \left(\frac{l + \frac{1}{2}}{4}\right)^2 \right) = 0 \quad (41)$$

Equation (41) is hypergeometric differential equation, so we must transform it to AIM-type equation by divide equation (41) with $(z(1-z))$, yields

$$p'' = p' \left(\frac{\left(2\alpha + \frac{1}{2}\right) - (2\alpha + 2\beta + 1)z}{z(1-z)} \right) + p \left(\frac{-(\alpha + \beta)^2 + \left(l + \frac{1}{2}\right)^2 \left(\frac{1}{4}\right)}{z(1-z)} \right) \quad (42)$$

From equation (42), we have

$$\lambda_0 = \frac{-\left(\left(2\delta + \frac{1}{2}\right) - (2\delta + 2\gamma + 1)z\right)}{z(1-z)} \quad (43a)$$

$$S_0 = \frac{\left((\alpha + \beta)^2 - \left(\frac{1}{2}l + \frac{1}{4}\right)^2\right)}{z(1-z)} \quad (43b)$$

To have this eigen value of equation, further iterations λ_i and S_i , which i is stated iteration. By using equation (5),

energy eigenvalues can be obtained, and by using software matlab, we have

$$\begin{aligned} \Delta_0 = s_0\lambda_1 - s_1\lambda_0 = 0 &\rightarrow (\alpha + \beta)^2 = \left(\frac{1}{2}l + \frac{1}{4}\right)^2 \\ \Delta_1 = s_1\lambda_2 - s_2\lambda_1 = 0 &\rightarrow (\alpha + \beta + 1)^2 = \left(\frac{1}{2}l + \frac{1}{4}\right)^2 \\ \Delta_2 = s_2\lambda_3 - s_3\lambda_2 = 0 &\rightarrow (\alpha + \beta + 2)^2 = \left(\frac{1}{2}l + \frac{1}{4}\right)^2 \end{aligned} \quad (44)$$

where $\varepsilon = E'$. from equation (44), can be generalized becomes:

$$l_{n_i} = 2\alpha + 2\beta + 2n_i - \frac{1}{2} \quad (45)$$

where n_i is orbital quantum number ($n_i = 1, 2, 3, \dots$)

The wavefunction of the system are obtained by comparing between equation (42) with equation (4), so we have

$$\begin{aligned} c = \frac{2\alpha - \frac{3}{2}}{2}, N = -1, t = \frac{2\beta + \frac{1}{2}}{2}, b = 1 \\ \sigma = \frac{2c + N + 3}{N + 2} = 2\alpha + \frac{1}{2}, p = \frac{(2c + 1)2t}{(N + 2)b} = 2\alpha + 2\beta \end{aligned} \quad (46)$$

From equation (5), we have

$$p_{n_i}(z) = (-1)^{n_i} C'(1)^{n_i} \left(2\delta + \frac{1}{2}\right)_{n_{i2}} {}_2F_1\left(-n_i, 2\alpha + 2\beta + n_i, 2\alpha + \frac{1}{2}, z\right) \quad (47)$$

By substitution Equation (47) into equation (40), we have

$$H = z^\alpha (1-z)^\beta (-1)^{n_i} C'(1)^{n_i} \left(2\delta + \frac{1}{2}\right)_{n_{i2}} {}_2F_1\left(-n_i, 2\alpha + 2\beta + n_i, 2\alpha + \frac{1}{2}, z\right)$$

Where $z = \cos^2 \theta$, so the angular wave function can be obtained, as follows

$$H(z) = (\cos\theta)^{2\alpha} (\sin\theta)^{2\beta} (-1)^{n_i} C'(1)^{n_i} \left(2\alpha + \frac{1}{2}\right)_{n_{i2}} F_1\left(-n_i, 2\alpha + 2\beta + n_i, 2\alpha + \frac{1}{2}, z\right) \quad (48)$$

C_{n_i} is angular normalization constant.

CONCLUSION

The Dirac equation for trigonometric Scarf II plus trigonometric Posch-Teller potential has already been settled by using the asymptotic iteration method. We obtained the relativistic energy in equation (32), the radial wavefunction in equation (36), the orbital quantum number in equation (45) and the angular wavefunction in equation (48).

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