ICMSE 2015
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RELATIVISTIC ENERGY AND THERMODYNAMICS PROPERTIES ANALYSIS DIRAC EQUATION OF Q-DEFORMED TRIGONOMETRIC POSCHL-TELLER POTENTIAL MODEL IN D DIMENSIONS USING ROMANOVSKI POLYNOMIAL

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ABSTRACT

We have apllied Romanovski polynomial to solve D-dimensional Dirac equation for q-deformed trigonometric Poschl-Teller potential. We reduced the D-dimensional Dirac equation for q-deformed Poschl-Teller potential to one dimensional Schrodinger type equation by suitable parameters, variable and wave function substitutions. The relativistic energy spectra were obtained numerically from the relativistic energy equation using MatLab programming and the relativistic wave functions were expressed in the Romanovski polynomial. In the non-relativistic limit, the relativistic energy equation was reduced to the non-relativistic energy. In the classical limit, the vibrational partition function, the vibrational specific heat, and the vibrational mean energy equations were generated from the non-relativistic energy equation. These thermodynamical properties of some diatomic molecules were then visualized with the help of error function and Matlab 2011

Key Word: Dirac Equation, q-Deformed Trigonometric Poschl-Teller Potential, Romanovski Polynomial, Thermodynamics Properties

INTRODUCTION

One of the important tasks of quantum mechanic relativistic is finding an accurate exact solution of Dirac equation for a certain potential. The bound state solutions of Dirac equation for non-central potentials, which are combination of radial shape invariance with angular shape invariant potentials, have been investigated by some authors using NU method (Hamzavi & Rajabi, 2013;Kurniawan,et.al,2015; Deta,et.al,2014), SUSY QM method (Sukumar,1985;Cari & Suparmi,2014;Suparmi & Cari, 2014), and Romanovski polynomial method (Cari, et.al, 2013; Suparmi ,et.al,2014).

It is known that for very limited potential, three dimensional radial Dirac equation is exactly solvable only for s-wave ($l = 0$). However, the three dimensional radial Dirac equation for the spherically symmetric potentials can not be solved analytically for $l \neq 0$ states because of the centrifugal term $\sim r^{-2}$(Ikhdair & Server,2008;Ikot & Akpabio,2010;Agboola,2011). The Schrodinger equation can only be solved approximately for different suitable approximation scheme. One of the suitable approximation scheme is conventionally proposed by Greene and Aldrich (Greene & Aldrich,1976). Furthermore, the extension in higher dimensional spaces for some physical problems is very important in some physics area. The D-dimensional non-relativistic and relativistic physical systems have been investigated by many authors, such as ring-shaped pseudoharmonic potential (Ikhdair & Server, 2008), isotropic harmonic oscillator plus inverse quadratic potential (Oyewumi,et.al,2003), Pseudoharmonic potential (Oyewumi,et.al,2008), Kratzer-Fues potential (Oyewumi,et.al,2005; Agboola, 2011), hydrogen atom (Al-Jaber,1998) ,modified Poschl-Teller potential (Agboola,2009), linierly energy dependent quadratic potential (Hassanabadi,et.al,2011), trigonometric scarf potential (Falaye & Oyewumi,2011), ring-shaped Kratzer potential (Ikhdair & Server,2007).

In this paper we will attempt to solve the D dimensional Dirac equation for a charged particle moving in a field governed by q-deformed trigonometric Poschl-Teller (Agboola, 2011) using Romanovski polynomial. The trigonometric Poschl-Teller potential is used to describe the motion of diatomic molecules (Agboola, 2011). Some of trigonometric non-central potentials are exactly solvable within the approximation of centrifugal term and their bound state solutions have been reported in the previous papers (Witten,1981; Gendenshtein,1983;Falaye & Oyewumi,2011;Ikhdair & Server,2007).

REVIEW OF DIRAC EQUATION IN D DIMENSION FOR CENTRAL POTENTIAL

The Dirac equation with the scalar potential $S(\vec{r})$ and magnitude of vector potential $V(\vec{r})$ is given as (Hu,et.al, 2010)

$$\{\vec{\alpha}.\vec{p}+\beta(M+S(\vec{r}))\}\psi(\vec{r})=\{E-V(\vec{r})\}\psi(\vec{r}) \quad (1)$$

where M is the relativistic mass of the particle, E is the total relativistic energy, and $\vec{p}$ is the three-dimensional momentum operator, $-i\nabla$

$$\vec{\alpha}=\begin{pmatrix}0 & \vec{\sigma}\\ \vec{\sigma} & 0\end{pmatrix},\ \beta=\begin{pmatrix}I & 0\\ 0 & -I\end{pmatrix} \quad (2)$$

with $\vec{\sigma}$ are the three-dimensional Pauli matrices and I is the 2×2 identity matrix. The potential in Eq.(1) is spherically symmetric potential, i.e. it does not only depend on the radial coordinate $r=|\vec{r}|$, and we have taken $\hbar=1$, $c=1$.

The Dirac equation expressed in Eq.(1) is invariant under spatial inversion and therefore its eigen states have definite parity. By writing the spinor in D dimension as

$$\psi(\vec{r})=\begin{pmatrix}\zeta(\vec{r})\\ \Omega(\vec{r})\end{pmatrix}=\begin{pmatrix}\dfrac{F_{nK}(r)}{r}Y_{jm}^{l}(\theta,\varphi)\\ i\dfrac{G_{nK}(r)}{r}Y_{jm}^{\bar{l}}(\theta,\varphi)\end{pmatrix}$$

$$\psi(\vec{r})=\begin{pmatrix}\zeta(\vec{r})\\ \Omega(\vec{r})\end{pmatrix}=\begin{pmatrix}\dfrac{F_{nK}(r)}{r^{\frac{D-1}{2}}}Y_{jm}^{l}(\theta,\varphi)\\ i\dfrac{G_{nK}(r)}{r^{\frac{D-1}{2}}}Y_{jm}^{\bar{l}}(\theta,\varphi)\end{pmatrix} \quad (3)$$

If we insert Eq.(2) and (3) into Eq. (1) and use matrices multiplication, we achieve

$$\vec{\sigma}.\vec{p}\,\Omega(\vec{r})=\{-M-S(\vec{r})+E-V(\vec{r})\}\zeta(\vec{r}) \quad (4)$$

$$\vec{\sigma}.\vec{p}\,\zeta(\vec{r})=\{M+S(\vec{r})+E-V(\vec{r})\}\Omega(\vec{r}) \quad (5)$$

In the exact spin symmetric case, when the scalar potential is equal to the magnitude of vector potential $S(\vec{r})=V(\vec{r})$, then from Eq.(4) and (5) we have

$$\vec{\sigma}.\vec{p}\,\frac{\vec{\sigma}.\vec{p}}{M+E}\zeta(\vec{r})=\{-M-2V(\vec{r})+E\}\zeta(\vec{r}) \quad (6)$$

By applying the Pauli matrices, it is simply shown that if $(\vec{\sigma}.\vec{p})(\vec{\sigma}.\vec{p})=p^2$, then Eq. (6) becomes

$$p^2+2V(\vec{r})(M+E)\zeta(\vec{r})=(E^2-M^2)\zeta(\vec{r}) \quad (7)$$

Since $p^2=-\Delta_D=-\nabla_D^2$ where ∇_D^2 is the hyper-spherical Laplacian which is stated as

$$\nabla_D^2=r^{1-D}\frac{\partial}{\partial r}\left\{r^{D-1}\frac{\partial}{\partial r}\right\}-\frac{L_D^2}{r^2};$$

$$L_D^2Y_{lm}=l(l+D-2)Y_{lm} \quad (8)$$

then we get the Dirac equation in D dimension by inserting Eq. (3) and (8) into Eq. (7) as

$$r^{1-D}\frac{\partial}{\partial r}\left\{r^{D-1}\frac{\partial}{\partial r}\left(\frac{F_{nK}(r)}{r^{\frac{D-1}{2}}}\right)\right\}-\frac{l(l+D-2)}{r^2}\left(\frac{F_{nK}(r)}{r^{\frac{D-1}{2}}}\right)$$
$$-\left(2V(\vec{r})(M+E)\right)\left(\frac{F_{nK}(r)}{r^{\frac{D-1}{2}}}\right)=-\left(E^2-M^2\right)\left(\frac{F_{nK}(r)}{r^{\frac{D-1}{2}}}\right) \quad (9)$$

that is simplified as

$$\frac{\partial^2F_{nK}(r)}{\partial r^2}-\frac{(l+\frac{D-1}{2})(l+\frac{D-3}{2})}{r^2}F_{nK}(r)$$
$$-\left(2V(\vec{r})(M+E)\right)F_{nK}(r)=-\left(E^2-M^2\right)F_{nK}(r) \quad (10)$$

To reduce the Dirac equation in Eq.(10) into one dimensional Schrodinger type equation we set $V\rightarrow\frac{1}{2}V$

$$\frac{\partial^2F_{nK}(r)}{\partial r^2}-\frac{(l+\frac{D-1}{2})(l+\frac{D-3}{2})}{r^2}F_{nK}(r)$$
$$-\left(V(\vec{r})(M+E)\right)F_{nK}(r)=-\left(E^2-M^2\right)F_{nK}(r) \quad (11)$$

Equation (11) is one dimensional Schrodinger type equation that can be solved using Romanovski polynomial.

REVIEW OF Q-DEFORMED TRIGONOMETRIC POSCHL-TELLER POTENTIAL

For spin and pseudospin symmetry the Dirac equation for q-deformed trigonometric Poschl-Teller potential within the q-deformed trigonometric cotangent plus cosecant type tensor reduces to Schrodinger-type equation therefore it can be solved using Romanovski polynomials. The q-deformed trigonometric Poschl-Teller potential given as

$$V(r,\theta_j) = t^2\left\{\frac{\eta(\eta-1)}{\sin_q^2 tr} - \frac{\lambda(\lambda-1)}{\cos_q^2 tr}\right\} \quad (12)$$

η, λ describe the depth of the trigonometric function well potential and are positives, t is a positive parameter which to control the width or the range of the potential well, q is the deformation of the potential, $q > 0$, t is the range of nucleon force, and $0 < r < \infty$.

ROMANOVSKI POLYNOMIAL

The Romanovski polynomials is used to solve the Dirac equation in the limit of spin symmetric and pseudospin symmetric cases since in the spin and pseudospin symmetric limited, the Dirac equations reduce to one dimensional Schrodinger-like equation. One dimensional Schrodinger equation of potential of interest reduces to the differential equation of Romanovski polynomial by appropriate variable and wave function substitutions. The one dimensional Schrodinger equation is given as

$$-\frac{\hbar^2}{2M}\frac{\partial^2\Psi(x)}{\partial x^2} + V(x)\Psi(x) = E\Psi(x) \quad (13)$$

where V(x) is an effective potential which is mostly shape invariant potential. By suitable variable substitution $x = f(s)$ Eq.(13) changes into generalized hypergeometric type equation expressed as

$$\frac{\partial^2\Psi(s)}{\partial s^2} + \frac{\tilde{\tau}(s)}{\sigma(s)}\frac{\partial\Psi(s)}{\partial s} + \frac{\tilde{\sigma}(s)}{\sigma^2(s)}\Psi(s) = 0 \quad (14)$$

with $\sigma(s)$ and $\tilde{\sigma}(s)$ are mostly polynomials of order two , $\tilde{\tau}(s)$ is polynomial of order one, s, $\sigma(s)$, $\tilde{\sigma}(s)$, and $\tilde{\tau}(s)$ can have any real or complex values (Cari, et.al, 2013; Suparmi ,et.al,2014).

Equation (14) is solved by variable separation method. By introducing new wave function in Eq. (15)

$$\Psi_n(r) = g_n(s) = (1+s^2)^{\frac{\beta}{2}} e^{\frac{-\alpha}{2}\tan^{-1}s} D_n^{(\beta,\alpha)}(s) \quad (15)$$

we obtain a hypergeometric type differential equation, which can be solved using finite Romanovski polynomials is expressed as ,(Cari, et.al, 2013; Suparmi ,et.al,2014)

$$\sigma(s)y''(s) + \tau(s)y'(s) + \lambda y(s) = 0 \quad (16)$$

with

$$\sigma(s) = as^2 + bs + c;\ \tau = fs + h$$
$$-\{n(n-1) + 2n(1-p)\} = \lambda = \lambda_n \quad (17)$$

and

$$y_n = R_n^{(p,q')}(s) = D_n^{(\beta,\alpha)}(s) \quad (18)$$

For Romanovski polynomials, the values of parameters in Eq. (10) are

$a = 1$, $b = 0$, $c = 1$, $f = 2(1\text{-}p)$ and $h = q'$ with $p > 0$ (19)

therefore Eq.(10) is rewritten as

$$(1+s^2)\frac{\partial^2 R_n^{(p,q')}}{\partial s^2} + \{2s(-p+1)+q'\}\frac{\partial R_n^{(p,q')}(s)}{\partial s} \quad (20)$$
$$-\{n(n-1)+2n(1-p)\}R_n^{(p,q')}(s) = 0 \quad (17)$$

Equation (19) which is obtained from Eq. (9) by applying the specific condition for Romanovski polynomials expressed in Eq. (13) is second order differential equation satisfied by Romanovski polynomials. Equation (10) is described in the textbook by Nikiforov-Uvarov where it is cast into self adjoint form and its weight function, w (s), satisfies Pearson differential equation

$$\frac{d(\sigma(s)w(s))}{ds} = \tau(s)w(s) \quad (21)$$

The weight function, $w(s)$, is obtained by solving the Pearson differential equation expressed in Eq. (15) and by applying condition in Eq. (11) and (13), given as

$$w^{(p,q')}(s) = (1+s^2)^{-p} e^{q'\tan^{-1}(s)} \quad (22)$$

The corresponding polynomials are classified according to the weight function, and are built up from the Rodrigues representation which is presented as

$$y_n = \frac{B_n}{w(s)}\frac{d^n}{ds^n}\left\{(as^2+bs+c)^n w(s)\right\} \quad (23)$$

with B_n is a normalization constant, and for $\sigma(s) > 0$ and w (s)>0, y_n (s)'s are normalized polynomials and are orthogonal with respect to the weight function $w(s)$ within a given interval (s_1, s_2) , which is expressed as

$$\int_{-\infty}^{\infty} w(s)y_n(s)y_{n'}(s)ds = \delta_{nn'} \quad (24)$$

This weight function in equation (22) first reported by Routh and then by Romanovski. The polynomial associated with equation (20) are named after Romanovski and will be denoted by $R_n^{(p,q)}(s)$. Due to the decrease of the weight function by s^{-2p}, integral of the type

$$\int_{-\infty}^{\infty} w^{(p,q')}R_n^{(p,q')}(s)R_{n'}^{(p,q')}(s)ds \quad (25)$$

will be convergent only if $n' + n < 2p - 1$ (26)

This means that only a finite number of Romanovski polynomials are orthogonal, and the orthogonality integral of the polynomial is expressed similar to the equation (24) where $y_n = R_n^{(p,q')}(s)$. The Romanovski polynomials obtained from Rodrigues formula expressed in equation (23) with the corresponding weight function in equation (22) is expressed as

$$R_n^{(p,q')}(s) = D_n^{(\beta,\alpha)}(s)$$
$$= \frac{1}{(1+s^2)^{-p}e^{q'\tan^{-1}(s)}}\frac{d^n}{ds^n}\left\{(1+s^2)^n(1+s^2)^{-p}e^{q'\tan^{-1}(s)}\right\} \quad (27)$$

If the wave function of the nth level in Eq.(15) is rewritten as

$$\Psi_n(r)=\frac{1}{\sqrt{\frac{df(s)}{ds}}}(1+s^2)^{\frac{-p}{2}}e^{\frac{q'}{2}\tan^{-1}(s)}R_n^{(p,q')}(s) \qquad (28)$$

then the orthogonality integral of the wave functions expressed in equation (28) gives rise to orthogonality integral of the finite Romanovski polynomials, that is given as

$$\int_0^\infty \Psi_n(r)\Psi_{n'}(r)dr=\int_{-\infty}^{\infty} w^{(p,q')}R_n^{(p,q')}(s)R_{n'}^{(p,q')}(s)ds \qquad (29)$$

In this case the values of p and q' are not n-dependence where n is the degree of polynomials. However, if either equation (24) or (26) is not fulfilled then the Romanovski polynomials is infinity.(Cari, et.al, 2013; Suparmi ,et.al,2014)

SOLUTION OF D DIMENSIONAL DIRAC EQUATION FOR Q-DEFORMED POSCHL-TELLER POTENTIAL

For radial q-deformed trigonometric Poschl-Teller potential which is given as

$$V(r)=t^2\left(\frac{\eta(\eta-1)}{\sin_q^2 tr}+\frac{\lambda(\lambda-1)}{\cos_q^2 tr}\right),$$

the D dimensional Dirac equation in Eq.(11) becomes

$$\frac{\partial^2 F_{nK}(r)}{\partial r^2}-\frac{(l+\frac{D-1}{2})(l+\frac{D-3}{2})}{r^2}F_{nK}(r)$$
$$-t^2\left(\frac{\eta(\eta-1)}{\sin_q^2 tr}+\frac{\lambda(\lambda-1)}{\cos_q^2 tr}\right)(M+E)F_{nK}(r)= \qquad (30)$$
$$-\left(E^2-M^2\right)F_{nK}(r)$$

$0<tr<\pi$ $\eta>1$; $\lambda>1$, η and λ are related to the depth of the potential, and t is to control the potential's width.

In order to solve the radial Dirac equation in Eq. (30), we use the approximation value for the centrifugal term as in Greene and Aldirch, and in Ikdhair, $\frac{1}{r^2}\cong t^2\left(\frac{1}{\sin_q^2 tr}\right)$. In the centrifugal approximation scheme, Eq. (30) becomes

$$\frac{\partial^2 F_{nK}(r)}{\partial r^2}-(l+\frac{D-1}{2})(l+\frac{D-3}{2})t^2\left(\frac{1}{\sin_q^2 tr}\right)F_{nK}(r)$$
$$-\left(\frac{\eta(\eta-1)}{\sin_q^2 tr}+\frac{\lambda(\lambda-1)}{\cos_q^2 tr}\right)(M+E)F_{nK}(r)= \qquad (31)$$
$$-\left(E^2-M^2\right)F_{nK}(r) \qquad (39)$$

$$\frac{\partial^2 F_{nK}(r)}{\partial r^2}-\left(\frac{t^2\eta(\eta-1)(M+E)+(l+\frac{D-1}{2})(l+\frac{D-3}{2})t^2}{\sin_q^2 tr}+\frac{t^2\lambda(\lambda-1)(M+E)}{\cos_q^2 tr}\right)F_{nK}(r)= \qquad (32)$$
$$-\left(E^2-M^2\right)F_{nK}(r) \qquad (40)$$

By setting

$$\eta(\eta-1)(M+E)t^2+(l+\frac{D-1}{2})(l+\frac{D-3}{2})t^2$$
$$=t^2\eta'(\eta'-1) \qquad (33)$$
$$\lambda(\lambda-1)(M+E)t^2=t^2\lambda'(\lambda'-1) \qquad (34)$$

and $\left(E^2-M^2\right)=t^2E'$ (35)

in Eq. (32) the it becomes

$$\frac{\partial^2 F_{nK}(r)}{\partial r^2}-\left(\frac{\eta(\eta-1)t^2}{\sin_q^2 tr}+\frac{\lambda(\lambda-1)t^2}{\cos_q^2 tr}\right)F_{nK}(r)=-E't^2F_{nK}(r) \qquad (36)$$

To reduce Eq. (36) into second order differential equation satisfied by Romanovski polynomial we set

$$\cos_q^2 tr=q\left(\frac{1+is}{2}\right) \qquad (37)$$

so we get

$$\sin_q tr=\sqrt{q-q\left(\frac{1+is}{2}\right)}=\sqrt{q\left(\frac{1-is}{2}\right)}$$ and

$$\frac{\partial^2}{\partial r^2}=-4t^2(1+s^2)\frac{\partial^2}{\partial s^2}-2t^2s\frac{\partial}{\partial s}$$

and Eq. (36) reduces to

$$-4(1+s^2)t^2\frac{\partial^2 F_{nK}}{\partial s^2}-4t^2 s\frac{dF_{n\kappa}}{ds}-\left(\frac{\eta'(\eta'-1)t^2 2(1+is)}{q(1+s^2)}+\frac{\lambda'(\lambda'-1)t^2 2(1-is)}{q(1+s^2)}\right)F_{nK}(r)=-E't^2F_{nK}(r) \quad (38)$$

that gives

$$(1+s^2)\frac{\partial^2 F_{nK}}{\partial s^2}+s\frac{dF_{n\kappa}}{ds}+\left(\frac{\eta'(\eta'-1)+\lambda'(\lambda'-1)}{2q(1+s^2)}+\frac{\eta'(\eta'-1)-\lambda'(\lambda'-1)}{2q(1+s^2)}is\right)F_{nK}(r)=\frac{E'}{4}F_{nK}(r) \quad (39)$$

By setting the solution of wave function in Eq. (39) as

$$F_{n\kappa}(r)=g_n(s)=(1+s^2)^{\frac{\beta}{2}}e^{\frac{-\alpha}{2}\tan^{-1}s}D_n^{(\beta,\alpha)}(s) \quad (40)$$

we obtain

$$(1+s^2)\frac{\partial^2 F_{nK}}{\partial s^2}+\{s(2\beta+1)-\alpha\}\frac{dF_{n\kappa}}{ds}-\left(\frac{2\beta\alpha s-s\alpha-\frac{\alpha^2}{2}+2\beta^2-2\beta-\frac{\eta'(\eta'-1)+\lambda'(\lambda'-1)}{q}-\frac{\eta'(\eta'-1)-\lambda'(\lambda'-1)}{q}is}{2(1+s^2)}\right)F_{nK}(r)-\left(\frac{E'}{4}-\beta^2\right)F_{nK}(r)=0 \quad (41)$$

Equation (41) will reduce into Romanovski polynomial differential by setting the term that its denominator with $2(1+s^2)$ to be zero and we obtain the conditions

$$2\beta\alpha-\alpha-\frac{\eta'(\eta'-1)-\lambda'(\lambda'-1)}{q}i=0;$$

$$-\frac{\alpha^2}{2}+2\beta^2-2\beta-\frac{\eta'(\eta'-1)+\lambda'(\lambda'-1)}{q}=0$$

and

$$(1+s^2)\frac{\partial^2 F_{nK}}{\partial s^2}+\{s(2\beta+1)-\alpha\}\frac{dF_{n\kappa}}{ds}-\left(\frac{E}{4}-\beta^2\right)F_{nK}(r)=0 \quad (42)$$

From those two equations in equation we obtain

$$(\beta-1/2)^2=\frac{\eta'(\eta'-1)+\lambda'(\lambda'-1)+1/2q\pm\sqrt{\{\eta'(\eta'-1)+\lambda'(\lambda'-1)+1/2q\}^2-\{\eta'(\eta'-1)-\lambda'(\lambda'-1)\}^2}}{4q} \quad (43)$$

and the value of β that has physical meaning is

$$(\beta-1/2)=\frac{1}{2}\left\{\sqrt{\frac{\{\eta'(\eta'-1)+1/4q\}}{q}}\pm\sqrt{\frac{\{\lambda'(\lambda'-1)+1/4q\}}{q}}\right\} \quad (44)$$

By comparing Eq.(16) and (42) we obtain

$$(2\beta+1)=2(1-p) \qquad p=(1/2-\beta)$$

$$\frac{E'}{4}-\beta^2=n(n-1)+2n(1-p)=n^2+2n\beta \quad (45)$$

By applying Eq. (44) into equation it is obtained the relativistic energy equation given as

$$\frac{E'}{4}=(\beta+n)^2=\left(\frac{1}{2}\left\{\sqrt{\frac{\{\eta'(\eta'-1)+1/4q\}}{q}}+\sqrt{\frac{\{\lambda'(\lambda'-1)+1/4q\}}{q}}+1\right\}+n\right)^2 \quad (46)$$

$$(E^2-M^2)=t^2E'=t^2\left(\left\{\sqrt{\frac{\{\eta'(\eta'-1)+1/4q\}}{q}}+\sqrt{\frac{\{\lambda'(\lambda'-1)+1/4q\}}{q}}+1\right\}+2n\right)^2 \quad (47)$$

with

$$\eta'(\eta'-1)=\eta(\eta-1)(M+E)+(l+\frac{D-1}{2})(l+\frac{D-3}{2})$$

and $\lambda'(\lambda'-1)=\lambda(\lambda-1)(M+E)$

The relativistic energy spectra that are obtained numerically from the relativistic energy equation is shown in Table 1.

Table 1. Relativistic energy for variation q

n_r	λ	M	η	d_0	t	l	D	q	$E_1\left(\frac{1}{fm}\right)$
1	2,5	10	0	1/12	0,8	2	2	0,25	26.156 13.898

								0,35	22.318 12.436
								0,45	20.098 11.692
								0,55	18.638 11.252
								0.95	15.720 10.513

The relativistic wave functions obtained , are given as

$$F_0 = (\sin tr)^{P'}(\cos tr)^B$$

$$F_1 = 2(-tP'\cot tr + tB\tan tr)(\sin tr)^{P'}(\cos tr)^B$$

$$F_2 = 2\begin{pmatrix}(-t^2P'\csc^2 tr - t^2B\sec^2 tr)+\\(-P't\cot tr + Bt\tan tr)+\\(-tP'\cot tr + tB\tan tr)^2\end{pmatrix}(\sin tr)^{P'}(\cos tr)^B$$

with

$$P' = \frac{1}{2} + \sqrt{\frac{\eta(\eta-1)(M+E)+(l+\frac{D-1}{2})(l+\frac{D-3}{2})}{q} + \frac{1}{4}};$$

$$B = \frac{1}{2} + \sqrt{\frac{\lambda(\lambda-1)(E+M)}{q} + \frac{1}{4}}$$

and are visually graphed using Mat Lab as shown in Figures (1).

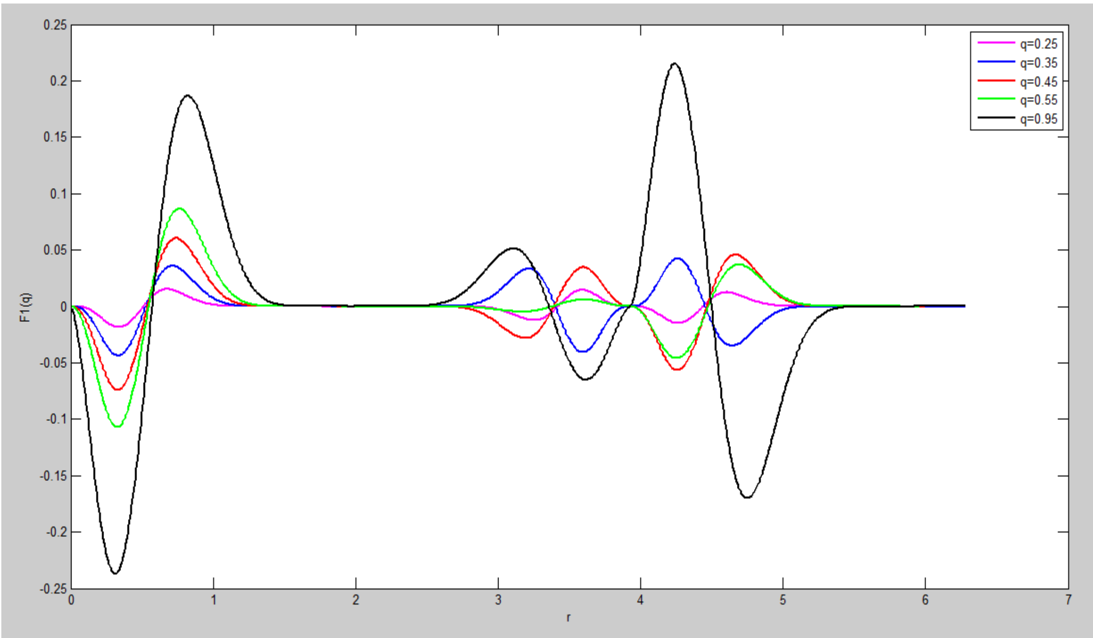

Figure 1. Radial wavefunctions
$F_1 = 2(-tP'\cot tr + tB\tan tr)(\sin tr)^{P'}(\cos tr)^B$ for variation q

THERMODYNAMICAL PROPERTIES

In non-relativistic condition, the relativistic energy equation expressed in Eq. (47) reduces into non-relativstic energy by taking $(M+E)\to 2\mu$ where μ is the non-relativistic mass, $(E-M)\to E_{NR}$, E_{NR} is the non-relativistic energy, then $E^2 - M^2 = (E+M)(E-M) = 2\mu E_{NR}$ and Eq. (47) becomes

$$2\mu E_{NR} = \left\{(l+\frac{D-1}{2})(l+\frac{D-3}{2})t^2 d_0\right\} + t^2\left(\left(\sqrt{\frac{\eta'_{Nr}(\eta'_{Nr}-1)}{q}+\frac{1}{4}}\right)+\left(\sqrt{\frac{\lambda'_{Nr}(\lambda'_{Nr}-1)}{q}+\frac{1}{4}}\right)+1+2n\right)^2 \quad (48)$$

with

$$\eta'_{Nr}(\eta'_{Nr}-1)=\eta(\eta-1)2\mu+(l+\frac{D-1}{2})(l+\frac{D-3}{2})$$

and $\lambda'_{Nr}(\lambda'_{Nr}-1)=\lambda(\lambda-1)2\mu$

Since d_0 is small then Eq. (48) reduces to

$$E_{NR}=\frac{t^2}{2\mu}\left(\begin{array}{l}\left(\sqrt{\frac{\eta'_{Nr}(\eta'_{Nr}-1)}{q}+\frac{1}{4}}\right)\\ +\left(\sqrt{\frac{\lambda'_{Nr}(\lambda'_{Nr}-1)}{q}+\frac{1}{4}}\right)\\ +1+2n\end{array}\right)^2 \quad (49)$$

In classical regimes (Ihdair,2013), the vibrational partition function, vibrational mean energy, and specific heat are obtained from the non-relativistic energy equation in Eq. (49). The vibrational partition function is defined as

$$Z(\zeta,\beta)=\sum_{n=0}^{\zeta}e^{-\beta E_{nl}},\beta=\frac{1}{kT} \quad (50)$$

k is Boltzman constant, E_{nl} is non-relativistic energy spectrum of the system. By rewriting Eq. (49) as

$$E_{NR}=\frac{2t^2}{\mu}(\xi-n)^2 \quad (51)$$

with

$$\xi=-\left\{\begin{array}{l}\frac{1}{2}\left(\sqrt{\frac{\eta'_{Nr}(\eta'_{Nr}-1)}{q}+\frac{1}{4}}\right)+\\ \frac{1}{2}\left(\sqrt{\frac{\lambda'_{Nr}(\lambda'_{Nr}-1)}{q}+\frac{1}{4}}\right)+\frac{1}{2}\end{array}\right\} \quad (52)$$

and by setting $\frac{1}{\delta^2}=\frac{2t^2\beta}{\mu}$ then from Eq.(50) and (51) we have

$$Z(\zeta,\beta)=\sum_{n=0}^{\frac{\xi}{\delta}}e^{-\left\{\frac{n-\xi}{\delta}\right\}^2}, \quad (53)$$

In the classical regime when the temperature, T, is high enough, then the value of ς is high , and β is small therefore Eq. (53) could be written into integral form as

$$Z(\xi,\beta)=\sum_{n=0}^{\frac{\xi}{\delta}}e^{-y^2}= \delta\int^{\frac{\xi}{\delta}}e^{-y^2}dy=\delta\frac{\sqrt{\pi}}{2}erf\left(\frac{\xi}{\delta}\right) \quad (54)$$

with $y=\frac{n-\xi}{\delta}$ and erf is the error function given as (Abramowitz & Stegun, 1972)

$$erf(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \quad (55)$$

The vibrational specific heat and the vibrational mean energy are defined as

$$U(\beta,\xi)=-\frac{\partial}{\partial\beta}\ln Z(\xi,\beta) \quad \text{and}$$

$$C=-\frac{\partial}{\partial T}U=-k\beta^2\frac{\partial}{\partial\beta}U \quad (56)$$

By using Eqs. (54-56) we obtain the vibrational mean energy equation given as

$$U(\beta,\xi)=-\frac{1}{\beta}\left(\begin{array}{l}-\frac{1}{2}+\\ \frac{\left(+\frac{1}{\sqrt{\pi}}\frac{\xi t\sqrt{2\beta}}{\sqrt{\mu}}\exp\left(-\frac{\xi^2t^2 2\beta}{\mu}\right)\right)}{erf\left(\frac{\xi t\sqrt{2\beta}}{\sqrt{\mu}}\right)}\end{array}\right) \quad (57)$$

and the vibrational specific heat which is obtained from Eqs. (56-57) is given as

$$C(\xi,\beta)=k\left(\begin{array}{l}\frac{1}{2}-\frac{\left(\frac{1}{\sqrt{\pi}}\frac{\xi t\sqrt{2\beta}}{\sqrt{\mu}}\exp\left(-\frac{\xi^2t^2 2\beta}{\mu}\right)\right)}{erf\left(\frac{\xi t\sqrt{2\beta}}{\sqrt{\mu}}\right)}\\ \left(\frac{1}{2}+\frac{\xi^2t^2 2\beta}{\mu}\right)-\\ \frac{\left(\frac{\xi^2t^2 2\beta}{2\sqrt{\pi}\mu}\exp\left(-2\frac{\xi^2t^2 2\beta}{\mu}\right)\right)}{\left(erf\left(\frac{\xi t\sqrt{2\beta}}{\sqrt{\mu}}\right)\right)^2}\end{array}\right) \quad (58)$$

CONCLUSION

We used Romanovski polynomial to solving the Dirac equation in D dimensions for q-deformed trigonometric Poschl-Teller potential. The q-deformed Poschl-Teller potential is used to describe the behavior of diatomic molecules.The radial part of D- dimensions of the Dirac equation reduces to one dimensional Schrodinger type equation in centrifugal approximation scheme. We obtained relativistic energy by using Romanovski polynomial. In the classical regime , some thermodynamics properties, such us the vibrational partition function, mean energy and specific heat are derived from the non-relativistic energy equation.

ACKNOWLEDGEMENT

This research was partly supported by Higher Education Project Grant with contract no. 698/UN27/PN/2015.

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